QUARTERLY PROGRESS REPORT

Study of Recirculating Gas Flow Fields in

the Base Region of Saturn-Class Vehicles

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Abstract

This report describes the work done under Contract NAS - 8-11485 during the period 1 November 1965 to 31 January 1966. The work during this period was directed towards an improved description of the reacting turbulent mixing layer found in the base region of large launch vehicles.

Characteristic times for the decay of turbulent eddies and for the transfer of energy through chemical reactions are computed and compared. It is found that, for an important energy-releasing reaction in the after-burning region, the turbulent mixing times are much longer than the reaction times. Thus, fluid-mechanical mixing may be the dominant process in the determination of the flame stand-off distance in the afterburning situation. Following the ideas of Proudian and Feldman, then, a procedure is presented for incorporating turbulent mixing in the afterburning jet problem. Numerical calculations have not yet been implemented, however.

The work presented in this report was accomplished by Graduate Research Assistant Mr. Jacques Nuyts under the direction of Prof. R. F. Hoglund.

1. <u>Characteristic Times in Reacting Turbulent Mixing</u>

In turbulence the characteristic time has been defined as the time it takes for the natural decay of eddies to transfer energy from one eddy corresponding to a wave number k to a smaller eddy corresponding to wave number $k + \Delta k$. This transfer of energy will continue to take place until the size of the eddy is so small that viscous dissipation and molecular diffusion become dominant.

It has been shown[1] that in the inertial convective subrange the characteristic time for turbulent decay is given by

$$\tau_T = [\, k^3 \, E\,(k)\,]^{-1/2}$$

where $E\,(k)$ is the turbulent energy spectrum. Since Kolmogorov's law $E\,(k) = \epsilon^{2/3}\, k^{-5/3}$ applies in this range τ_T can be expressed as

$$\tau_T = [\, \epsilon^{1/2}\, k\,]^{-2/3} \tag{1}$$

Also, as was previously shown, if a spectrum $G\,(k)$ is defined for the concentration field of a reactant, the characteristic time corresponding to the reactive energy transfer rate is

$$\tau_R = [\, k\, c^2\, G\,(k)\,]^{-1/2}$$

where C is the reaction rate constant. Again, expressing G (k) in the inertial subrange

$$G(k) = B k^{-5/3} \exp [3 C \epsilon^{-1/3} k^{-2/3}]$$

and

$$\tau_R = B^{-1/2} C^{-1} k^{1/3} \exp [-\frac{3}{2} C \epsilon^{-1/3} k^{-2/3}] \qquad (2)$$

where B is a constant to be defined later and ϵ is the total rate of energy dissipation.

These times are locally defined. To find the time corresponding to the "dissipation" of a given inhomogeneity in the flow field τ_T has to be summed over an interval of wave numbers.

It will be assumed that the original scale for inhomogeneities introduced in the flow corresponds to the size of the largest eddies in the boundary region of the jet.

It is further assumed that the original scale defined above can be approximated by ℓ_e $1/k_e$, where k_e corresponds to the maximum of the energy spectrum.

A reactant element will be considered to be thoroughly mixed with the fluid in the jet core at a point where the large eddies which captured the element at the jet boundary have decayed into the range of molecular diffusion. This point corresponds to the wave number "k_d". Using the Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$, where ν is the kinematic viscosity,
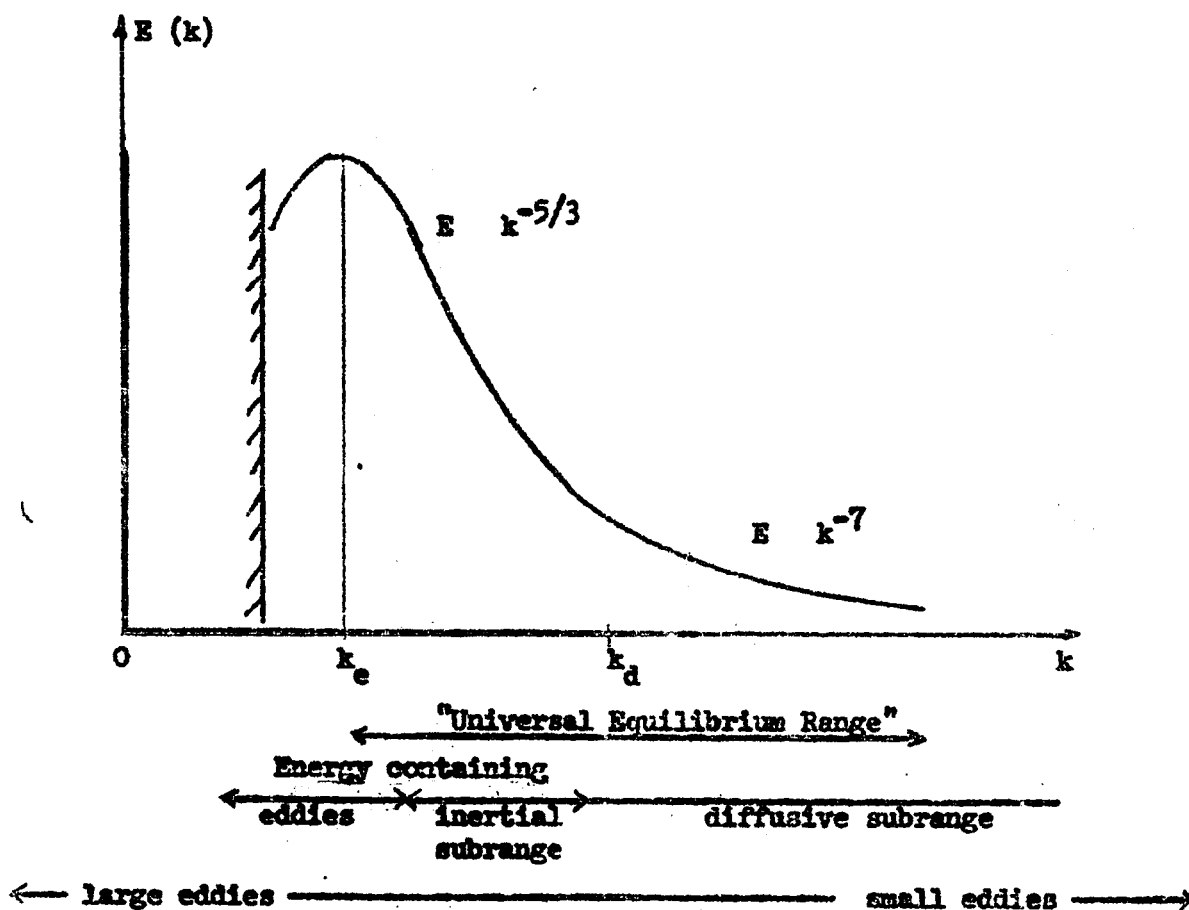
$$k_d = 1/\eta$$

Figure 1. Energy Spectrum

Hence the time it takes turbulence to mix any new element introduced into the mean flow can be expressed as

$$\tau_T = \alpha \int_{k_e}^{k_d} \frac{d\tau}{dk} \; dk$$

where α is a constant of order unity. Substituting equation (1)

$$\tau_T = \frac{3}{2} \alpha \; \epsilon^{-1/3} \; [\; k_e^{-2/3} - k_d^{-2/3} \;]$$

For steady, homogeneous, isotropic turbulence the following relations hold.

$$\frac{\ell_e}{\eta} = 15^{-3/4} \, A \, Re_\lambda^{3/2} = 15^{-3/4} \, A \left(\frac{u' \lambda_g}{\nu} \right)^{3/2}$$

$$\lambda_g = u' \left(\frac{15 \nu}{\epsilon} \right)^{1/2}$$

where λ_g is the dissipation scale

                 u' is the RMS turbulent velocity

                 A is a constant of order unity

Thus τ_T can be written as

$$\tau_T = \frac{3\alpha}{2} \epsilon^{-1/3} \eta^{2/3} \left[15^{-1/2} A^{2/3} \frac{u' \lambda_g}{\nu} - 1 \right]$$

but for large Reynolds number $k_e \lll k_d$, or $\ell_e \ggg \eta$, thus $k_d^{-2/3}$ is negligible compared to $k_e^{-2/2}$, and τ_T becomes

$$\tau_T = \left(\frac{3\alpha}{2} A^{2/3} \right) \frac{(u')^2}{\epsilon} = K_T \frac{u'^2}{\epsilon} \tag{3}$$

where K_T is a constant of order unity.

It is seen that τ_T given by (3) is only a rough approximation of the effective time for turbulent mixing of the engulfed fluid. First of all the spectrum of kinetic turbulent energy has been integrated not only over the inertial but also over the diffusive subrange. In the latter range $E(k) \sim k^{-7}$ instead of $k^{-5/3}$, which was used. The choice

of k_e and k_d appear arbitrary but if the physical picture is correct the resulting τ_T will be an upper limit of the effective time of mixing.

A similar procedure will be followed for the time of energy transfer by chemical reactions. As stated previously $G(k)$ as defined applies only to the inertial subrange. The relationship for larger wave numbers as given by Corrsin[1] and Batchelor[2] differ significantly.

Integrating as before from k_e to k_d with $k_d \ggg k_e$

$$\tau_R = \beta \int_{k_e}^{k_d} \frac{d\tau_R}{dk} \, dk$$

where β is of order unity. Neglecting terms of order $\left(\frac{k_e}{k_d} \right)^{1/3}$ gives

$$\tau_R = \beta \, B^{-1/2} \, c^{-1} \, k_d^{1/3} \, \exp\left[-\frac{3}{2} c \, \epsilon^{-1/3} \, k_d^{-2/3} \right]$$

But $B \quad \overline{\theta^2} \, k^{2/3}$ where $\overline{\theta^2}$ is the mean square concentration. This is obtained by dimensional analysis, expanding the exponential term for small $[c \, \epsilon^{-1/3} \, k_d^{-2/3}]$. Substituting B and k_d in τ_R

$$\tau_R = \beta \, B^{-1/2} \, c^{-1} \, \left(\frac{\epsilon}{\nu^3} \right)^{1/2} \, \exp\left[-\frac{3}{2} c \, \epsilon^{-1/3} \left(\frac{\epsilon}{\nu^3} \right)^{-1/6} \right]$$

$$= \frac{\beta'}{\theta' c} \, \exp\left[-\frac{3}{2} c \, \epsilon^{-1/2} \, \nu^{1/2} \right]$$

where θ' is the RMS of concentration. Expanding for small $[c \, \epsilon^{-1/2} \, \nu^{1/2}]$ gives

$$\tau_R = \frac{2}{3} \beta' \frac{\epsilon^{1/2}}{C^2 \nu^{1/2} \theta'} = K_R \frac{\epsilon^{1/2}}{C^2 \nu^{1/2} \theta'}$$

where K_R is a constant of order unity. Therefore τ_R is a function of two parameters ϵ and ν describing an isotropic turbulent flow in the inertial subrange and of the two chemical parameters the reaction rate constant C and concentration, θ'.

It is now possible to obtain some representative values for the respective times. Data for a characteristic rocket exhaust jet as given by Feigenbutz[4] have been used. The conditions given below are at $x = 0$ on the centerline of the jet (subscript J).

Altitude	15.5×10^3 ft.	50×10^3 ft.
T_J	1750 °K	1230 °K
ρ_J	5.5×10^{-3} lbs/ft^3	1.62×10^{-3} lbs/ft^3
u_J	9.333 fps	9.850 fps
$[c_{H_2}]_J$	0.013	0.013

Data were also used at the boundary of the jet for $x = 18\ r_J$ and $76\ r_J$.

Altitude, ft.	15.5×10^3	15.5×10^3	50×10^3
x/r_J	18	76	16
r_B/r_J	1	1.25	1
T/T_J	1.5	1.7	1.81
ρ/ρ_J	0.75	0.75	0.57
u/u_J	0.68	0.48	0.65
$[c_{H_2}]/[c_{H_2}]_J$	0.45	0.1	0.44

The reaction considered

$$OH + H_2 \overset{\textcircled{1}}{\rightleftharpoons} H_2) + H$$
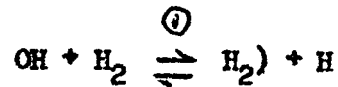
is one of the most important for energy release. According to Kaufman and Delgreco[5]

$$k_1 = 10^{13.8} (e^{-\frac{5900}{RT}}) \qquad 300 < T < 2000 \ ^\circ K$$

The reaction rate is then

$$C = \rho \, k_1$$

General data:

$$u = 2.7 \times 10^{-5} \text{ lbs/ft sec}$$

x/r_J	16	18	76
ϵ/u'^3	6.16	6.93	28.2

(The latter values were derived from the values of the dissipation length scale $\quad \ell_e = \frac{1}{3\sqrt{3}} \frac{u'^3}{\epsilon} (\frac{x - x_0}{2})\quad$ in Townsend[6].)

With these data the following values of τ_T and τ_R were obtained

Altitude	x/r_J	τ_T (sec)	τ_R (sec)
15.5×10^3	18	$K_T \ 2.28 \times 10^{-5}$	$K_R \ 3.96 \times 10^{-13}$
	76	$K_T \ 0.792 \times 10^{-5}$	$K_R \ 15.1 \times 10^{-13}$
50×10^3	16	$K_T \ 2.54 \times 10^{-5}$	$K_R \ 5.43 \times 10^{-13}$

It is to be noted that the values of τ_T obtained here agree fairly well with usual ignition delay times obtained in the literature for turbulent flame combustion. The magnitude of the ratio τ_T/τ_R indicates that the delaying factor in the chemical heat release (at least for the chosen reaction) is not the chemistry itself. It is instead the delay in turbulent mixing of reactants. The reaction rate constant as well as θ' may differ markedly for another reaction. A new model of turbulent mixing is introduced by Proudian and Feldman[7]. Let us consider a jet flow with a mean turbulent boundary as given in Figure 2.
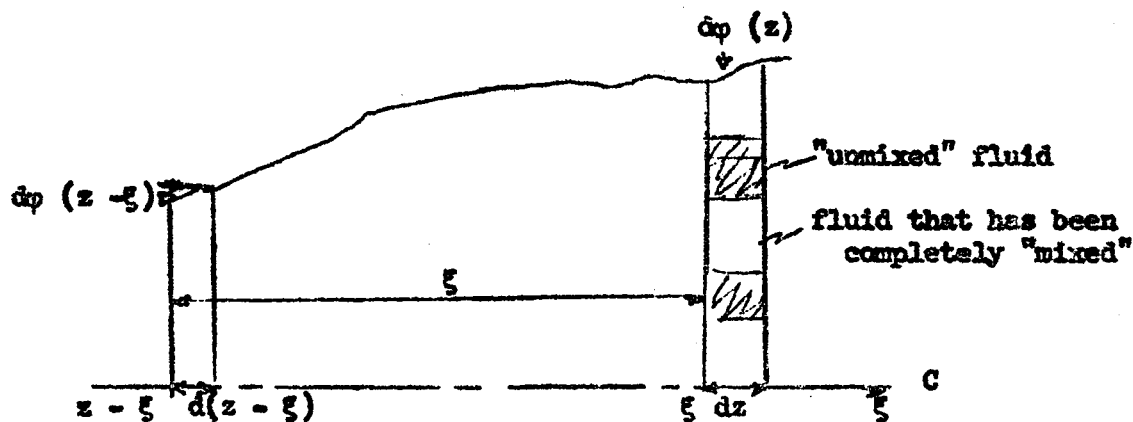

Figure 2. Jet Flow with Mean Turbulent Boundary

The mixing of surrounding gas with fluid in the turbulent core takes place first by means of the action of large eddies which convect large fluid elements (large with respect to the Kolmogorov scale). At the same time these elements are distorted by the shearing effects of the smaller eddy motion. This process occurs until the scale of the inhomogeneities

has been reduced to the point that molecular diffusion "mixes" the engulfed fluid. This mixing takes place over a length $\xi_T = K_T \frac{u'^3}{\epsilon}$, where u' is defined at the boundary at the station $(z - \xi_T)$. An element of fluid entering the fluid jet at $(z - \xi_T)$ will be considered to be part of the jet core at (z), where mixing is completed and reaction can occur. The reaction will occur over a length given approximately by

$$\xi_R = K_R \left(\frac{\epsilon}{\nu} \right)^{1/2} \frac{u'}{c^2 \theta'}$$

Here u' should be evaluated at (z) instead of $(z - \xi_T)$ but ξ_T is small enough that it is considered invariant in that interval. Thus ξ_R is nearly invariant. From the values of the ratio τ_T/τ_R, chemical reaction will occur almost as soon as mixing is completed. The corresponding energy release then will take place instantly.

The turbulent core may be resolved into two components: (1) a part which has not been mixed on a molecular basis with the main flow - this portion of the engulfed fluid maintains its original properties at station (z) this corresponds to the elements entering the jet after $(z - \xi_T)$; (2) a homogeneous component which consists of the initial jet fluid and the engulfed fluid products after the reaction.

Thus any fluid element entering the jet will mix and react chemically after what is called a lag-time or lag-distance given by

$$\tau = \tau_R + \tau_T = K_T \frac{u'^2}{\epsilon} + K_R \left(\frac{\epsilon}{\nu} \right)^{1/2} \frac{1}{c^2 \theta'}$$

$$\xi = \xi_R + \xi_T = K_T \frac{u'^3}{\varepsilon} + K_R \left(\frac{\varepsilon}{\nu} \right)^{1/2} \frac{u'}{c^2 \theta'}$$

where $\quad \tau_R \lll \tau_T \, , \qquad \xi_R \lll \xi_T \, .$

The external fluid entering the core at $(z - \xi)$ will travel over the distance ξ before it can be considered as part of the mixed fluid. Hence the change in jet properties are controlled over a distance dz by the fluid entering the jet in $d(z - \xi)$. Writing the equations of mass, momentum and energy conservation an attempt is made to describe the jet mixing region for two dimensional or axisymmetric flows.

An axially symmetric flow is considered in which the boundary layer approximations are valid according to the general mixing theory. In particular it is assumed that there is no pressure gradient in the mixing region. Let x be axial distance, y the radial distance and u and v the respective velocity components. Barred values are time averages with usual turbulence characteristics.

The general set of equations become:

1) Mass conservation

$$\frac{\partial \overline{\rho} \, \overline{u}}{\partial x} + \frac{\partial \overline{\rho \, v}}{\partial y} + \frac{\overline{\rho \, v}}{y} = 0$$

2) Momentum conservation

$$\overline{\rho} \, \overline{u} \frac{\partial \overline{u}}{\partial x} + \overline{\rho \, v} \frac{\partial \overline{u}}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} \left[y \, \varepsilon \, \overline{\rho} \frac{\partial \overline{u}}{\partial y} \right]$$

where ε is the eddy viscosity which is assumed to be independent of y.

3) Species conservation

$$\bar{\rho}\,\bar{u}\frac{\partial C_i}{\partial x} + \overline{\rho\,v}\frac{\partial C_i}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left[\, y\,\bar{\rho}\,D_{12}\frac{\partial C_i}{\partial y}\,\right] + \overset{o}{w}_i$$

where D_{12} is the binary diffusion coefficient (assuming that only one coefficient can be chosen for the species present) and $\overset{o}{w}_i$ is the rate of production of species i.

4) Energy Equation

$$\bar{\rho}\,\bar{u}\frac{\partial \bar{h}}{\partial x} + \overline{\rho\,v}\frac{\partial \bar{h}}{\partial y} = \frac{1}{y}\frac{\partial}{\partial y}\left[\, C_{p_f}\,\bar{\rho}\,\epsilon\,y\frac{\partial \bar{T}}{\partial y} + \bar{\rho}\,y\,D_{12}\sum_i h_i\frac{\partial C_i}{\partial y}\,\right]$$

$$+ \epsilon\,\bar{\rho}\left(\frac{\partial \bar{u}}{\partial y}\right)^2$$

where $\qquad \bar{h} = C_{p_f}\bar{T}$

$$h_i = \overset{o}{h}_i + \int_0^T C_{p_i}\,dT$$

$$C_{p_f} = \sum_i C_i\,C_{p_i}$$

5) Perfect gas equation of state

$$\bar{p} = \frac{\bar{\rho}}{M}R_o\bar{T}$$

$$p_i = \frac{\rho_i}{M_i}R_o\bar{T} \qquad\qquad C_i = \frac{\rho_i}{\bar{\rho}}$$

$$\bar{p} = \sum p_i$$

also $\quad \sum_i C_i = 1 \quad$ and $\quad \sum_i \overset{o}{w}_i = 0$

Let φ_1 be the amount of element i that enters the turbulent jet from the atmosphere and ξ be the distance it travels before it is "mixed". The element has a velocity V and therefore its momentum is added to that of the jet. $\varphi_1 = C_i' \varphi$ where C_i' is the concentration of species i in the surrounding air. In general $C_i' = 0$ except for O_2, O, N_2, N. C_i' will be a function of altitude. φ is a function of x and y. Should the boundary of the jet be known, $y_b(x)$, φ would be a function of x only by assuming that φ is a function of y at any station x.

The differential equations now become

1) $\dfrac{\partial \overline{\rho}\,\overline{u}}{\partial x} + \dfrac{\partial \overline{\rho}\,\overline{V}}{\partial y} + \dfrac{\overline{\rho}\,\overline{V}}{y} = \varphi$

2) $\overline{\rho}\,\overline{u}\,\dfrac{\partial \overline{u}}{\partial x} + \overline{\rho\,V}\dfrac{\partial \overline{u}}{\partial y} = \dfrac{1}{y}\dfrac{\partial}{\partial y}[y\,\epsilon\,\overline{\rho}\,\dfrac{\partial \overline{u}}{\partial y}] + \varphi\,V$

3) $\overline{\rho}\,\overline{u}\,\dfrac{\partial C_1}{\partial x} + \overline{\rho\,V}\dfrac{\partial C_1}{\partial y} = \dfrac{1}{y}\dfrac{\partial}{\partial y}[y\,\overline{\rho}\,D_{12}\dfrac{\partial C_1}{\partial y}] + \dot{w}_1 + \varphi_1$

4) $\overline{\rho}\,\overline{u}\,\dfrac{\partial \overline{h}}{\partial x} + \overline{\rho\,V}\dfrac{\partial \overline{h}}{\partial y} = \dfrac{1}{y}\dfrac{\partial}{\partial y}[C_{p_f}\,\overline{\rho}\,\epsilon\,y\,\dfrac{\partial \overline{T}}{\partial y} + \overline{\rho}\,y\,D_{12}\sum_i h_i\dfrac{\partial C_1}{\partial y}]$

$$+ \;\epsilon\,\overline{\rho}\,(\dfrac{\partial \overline{u}}{\partial y})^2 + \varphi_\xi\,H_e$$

where H_e is the total enthalpy of the engulfed fluid $H_e = h_e + \dfrac{V^2}{2}$. Note that due to the mixing lag φ in equation (4) is evaluated at $x - \xi$ while the other parameters are evaluated at x. This expresses the fact that after a time τ the newly engulfed fluid will be able to release its energy to the flow field: $\varphi_\xi = \varphi(x - \xi)$. A more

accurate picture would be to sum over the whole downstream distance:

$$\varphi_\xi = \varphi (x - \xi) \int_0^x \frac{\partial \alpha}{\partial z'} \, dz'$$

where α now represents the portion of the fluid that is already mixed with the core at station x'. But if the mixing (which in fact is gradual) takes place over a small length compared to the range over which the inviscid flow properties change we can take α as a δ - function peaking at $x - \xi$.

$$\frac{\partial \alpha}{\partial z} = \delta (z - \xi)$$

and then take

$$\varphi_\xi = \varphi (x - \xi)$$

The description of ξ and $\varphi (x)$ then completes the model and a numerical solution can be sought. In doing so a mean boundary is first computed from laminar flow $y_b (x)$. A function $\varphi (x)$ for the engulfment of external fluid is then assumed.

2. Work Planned for Next Reporting Period

Due to the graduation (with an M.S. degree) of Mr. Nuyts and the beginning of a new semester, the burden of work during the next reporting period will fall to Mr. H. F. Nelson, Graduate Research Instructor. Under the direction of Prof. Robert Goulard, Mr. Nelson will study the basis for and applicability of statistical treatments of molecular band models for radiation.

3. References

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